

# Prestazioni dinamiche di strumenti di misura

## Parte II

- La funzione di trasferimento
- La funzione di trasferimento sinusoidale
- Condizioni ideali di funzionamento di uno strumento
- Diagrammi logaritmici di Bode

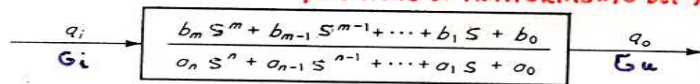
ESEMPIO

$$a_n \frac{d^n q_o}{dt^n} + a_{n-1} \frac{d^{n-1} q_o}{dt^{n-1}} + \dots + a_1 \frac{dq_o}{dt} + a_0 q_o = b_m \frac{d^m q_i}{dt^m} + b_{m-1} \frac{d^{m-1} q_i}{dt^{m-1}} + \dots + b_1 \frac{dq_i}{dt} + b_0 q_i$$

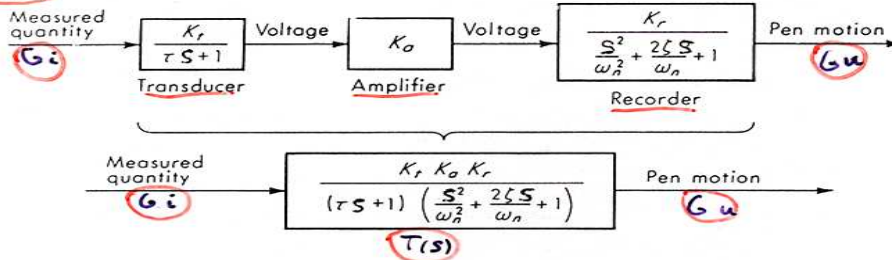
L. -TRASF.

$$\frac{q_o(s)}{q_i(s)} \triangleq \frac{q_o}{q_i}(s) \triangleq \frac{b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0} \quad (\text{CONDIZIONI INIZIALI NULLE})$$

FUNZIONE DI TRASFERIMENTO DEL SISTEMA



ESEMPIO



RAPPRESENTAZIONE DELLA FUNZIONE DI TRASFERIMENTO SINUSOIDALE

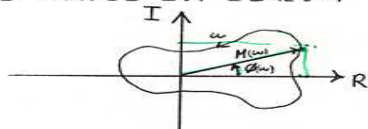
A) 2 DIAGRAMMI CARTESIANI  $R(\omega)$  E  $I(\omega)$



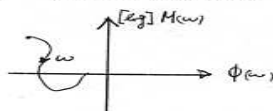
B) 2 DIAGRAMMI CARTESIANI  $M(\omega)$  E  $\phi(\omega)$



C) DIAGRAMMA DI NYQUIST DI  $M(\omega)$  E  $\phi(\omega)$  CON  $\omega$  PARAMETRO CORRENTE LUNGO LA CURVA



D) DIAGRAMMA DI NICHOLS CON  $\phi(\omega)$  IN ASCISSA E  $[20] M(\omega)$  IN ORDINATA CON  $\omega$  PARAMETRO CORRENTE LUNGO LA CURVA.



E) DIAGRAMMA LOGARITMICO DI BODE COL  $20 \log M(\omega)$  E LA FASE  $\phi(\omega)$  IN FUNZIONE DEL LOGARITMO DI  $\omega$   $[LN M - LG_2 \omega$  O  $20 \cdot LOG M - LOG \omega]$



ESEMPIO

PRECEDENTE:  $\frac{d g_v}{dt} + 6 g_v = 8$  ;  $g_v(0^+) = 2$

$T(s) = G_v^*(s)$  SE  $g_i = \delta(t)$  E  $g_v(0^+) = 0$

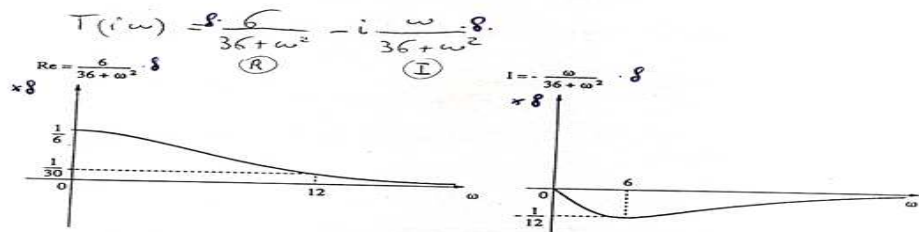
$\Rightarrow \frac{d g_v}{dt} + 6 g_v = 8 \delta(t)$  ;  $g_v(0^+) = 0$

$\Rightarrow s G_v^*(s) + 6 G_v^*(s) = 1 \cdot 8 \Rightarrow T(s) = G_v^*(s) = \frac{1}{s+6} \cdot 8$

FUNZIONE DI TRASFERIMENTO SINUSOIDALE:

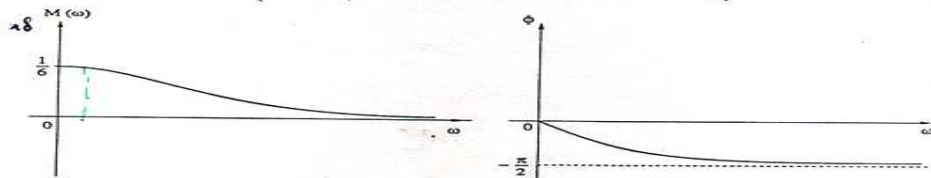
$$T(i\omega) = 8 \cdot \frac{1}{6+i\omega} \left( \frac{6-i\omega}{6-i\omega} \right) = 8 \cdot \frac{6-i\omega}{36+\omega^2}$$

A)

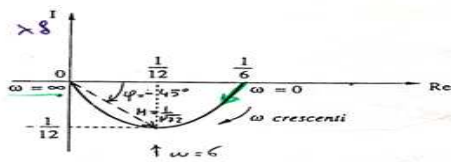


B)

$$M(\omega) = \sqrt{R^2 + I^2} = 8 \cdot \sqrt{\frac{36+\omega^2}{(36+\omega^2)^2}} = 8 \cdot \frac{1}{\sqrt{36+\omega^2}} ; \quad \phi = \text{ARCTG} \frac{I}{R} = \text{ARCTG} \left( \frac{-\omega}{6} \right)$$

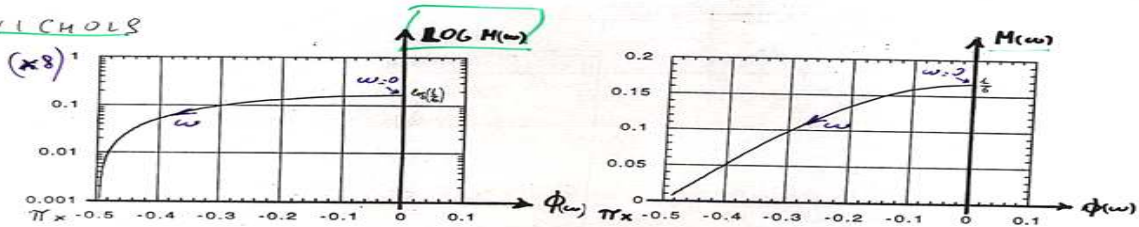


c) NYQUIST



$$T(i\omega) = \frac{6 - i\omega}{36 + \omega^2}$$

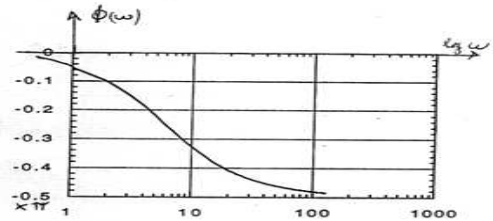
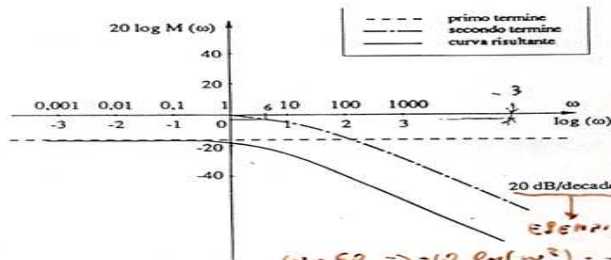
d) NICHOLS



e) BODE

(x8)

$$20 \log M = 20 \log (36 + \omega^2)^{-\frac{1}{2}} = -10 \log 36 - 10 \log (1 + \frac{\omega^2}{36})$$



ESEMPLO

$$\left. \begin{aligned} \omega = 60 &\Rightarrow -10 \log \left( \frac{60^2}{36} \right) = -10 \log 100 = -20 \\ \omega = 600 &\Rightarrow -10 \log \left( \frac{600^2}{36} \right) = -10 \log 10000 = -40 \end{aligned} \right\} 20 \text{ dB/decade}$$



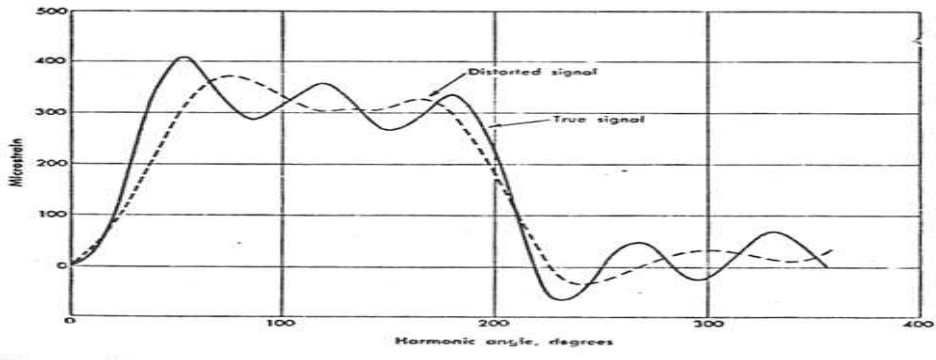
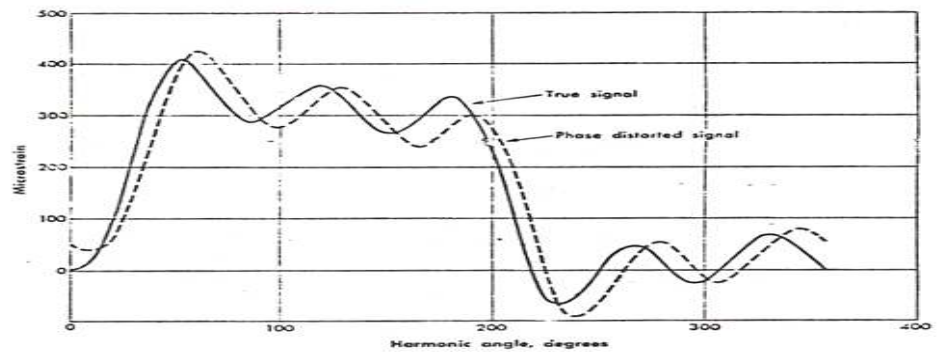
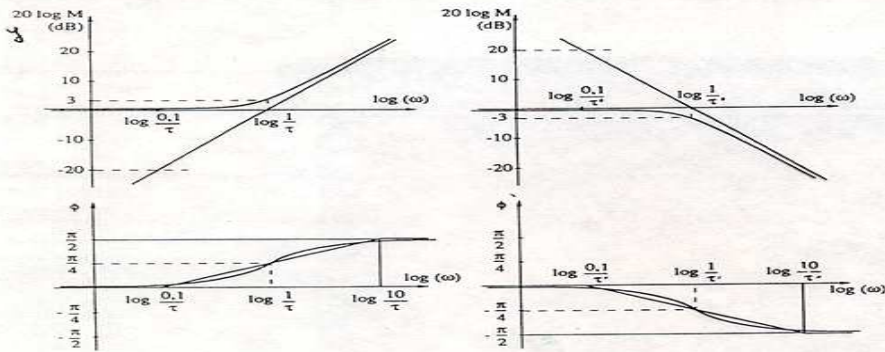


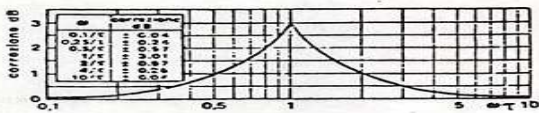
Fig. 3.16 Effect of poor frequency response on recording of strain-time relation shown in Fig. 3.12(b).

③:  $(1 + i\omega\tau)$  oppure  $(1 + i\omega\tau')^{-1}$

$20 \log M = 20 \log \sqrt{1 + (\omega\tau)^2}$  ;  $\Phi = \text{arctg } \omega\tau$   
 $20 \log M' = -20 \log \sqrt{1 + (\omega\tau')^2}$  ;  $\Phi' = \text{arctg } \omega\tau'$

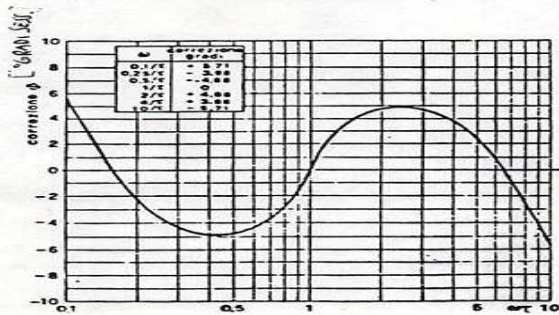


Rappresentazione di Bode di termini della forma  $(1+i\omega\tau)^{\pm 1}$

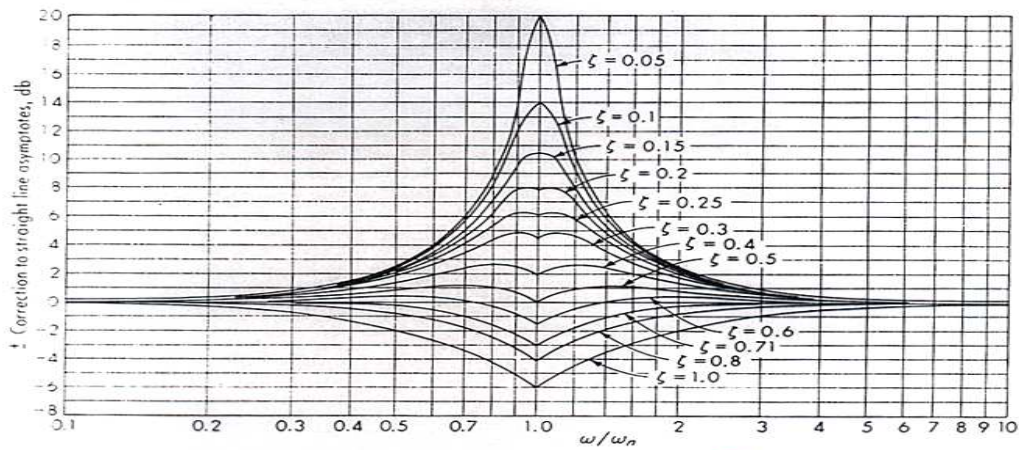


per  $\omega\tau \gg 1$  e  $\omega\tau' \gg 1$  si ha:  $\alpha \quad \infty$   
 $20 \log_{10} M = 20 \log \tau + 20 \log \omega$   
 $20 \log M' = -20 \log \tau' - 20 \log \omega$

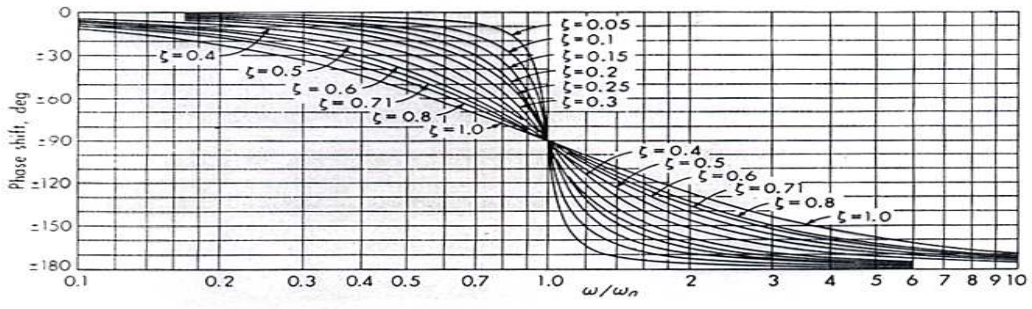
per  $\omega\tau \ll 1$  e  $\omega\tau' \ll 1$  si ha:  
 $\log M = \log M' = 0$



punti di rottura  
 $\varphi = 0 \Rightarrow \log \omega = -\log \tau = \log \frac{1}{\tau}$   
 $\log \omega = -\log \tau' = \log \frac{1}{\tau'}$



Second-order-system amplitude-ratio corrections. *(FUNG - II O.R.D. AL DENOMIN.)*



Second-order-system phase angle.

## PROCEDIMENTO

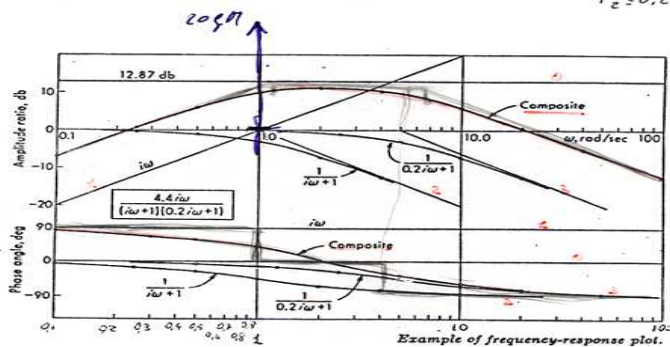
- 1) SI TRACCIA LA RETTA PARALLELA ALL'ASSE DELLE ASCISSE DI VALORE  $M = 20 \log_{10} K$ .
- 2) SI RIPORTANO SULL'ASSE DELLE ASCISSE I VALORI DEI PUNTI DI ROTTURA  $\log_{10} 1/T_1$  PER I TERMINI DEL PRIMO ORDINE E I VALORI  $\log_{10} \omega_{nc}$  PER QUELLI DEL II° ORDINE.
- 3) SI TRACCIAMO LE CURVE ASINTOTICHE DEI TERMINI DEL I° E II° ORDINE.
- 4) SI APPORTANO LE RELATIVE CORREZIONI AI TERMINI.
- 5) SI SOMMIAMO PUNTO PER PUNTO I VALORI DELLE SINGOLE CURVE E SI OTTIENE LA CURVA RISULTANTE.

$$\frac{g_v}{g_i}(i\omega) = \frac{4,4 i\omega}{(i\omega+1)(0,2i\omega+1)}$$

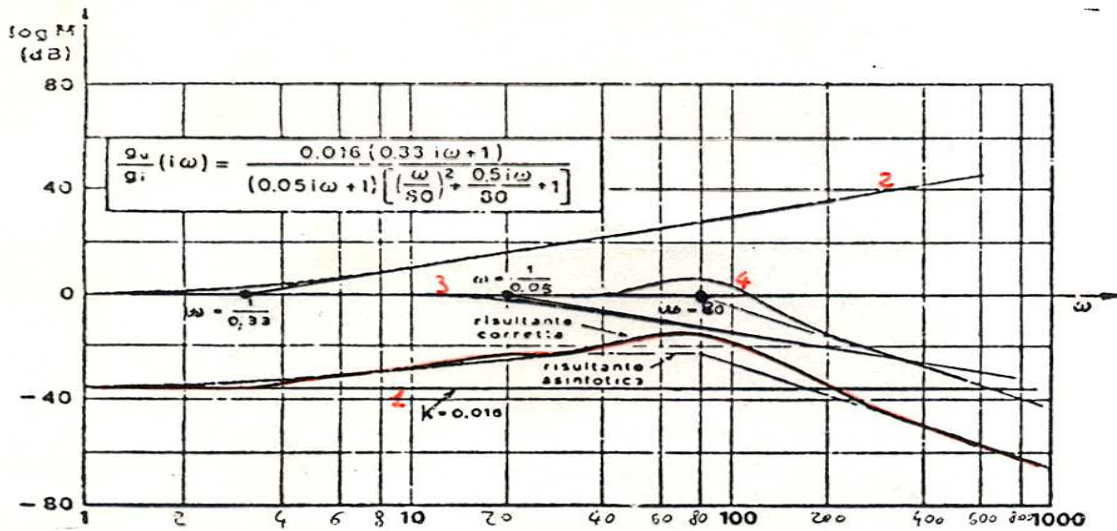
$$20 \log_{10} 4,4 = 12,87 \text{ dB}$$

$$T_1 = 1 \quad \sqrt{T_1} = 1 \text{ RAD/S}$$

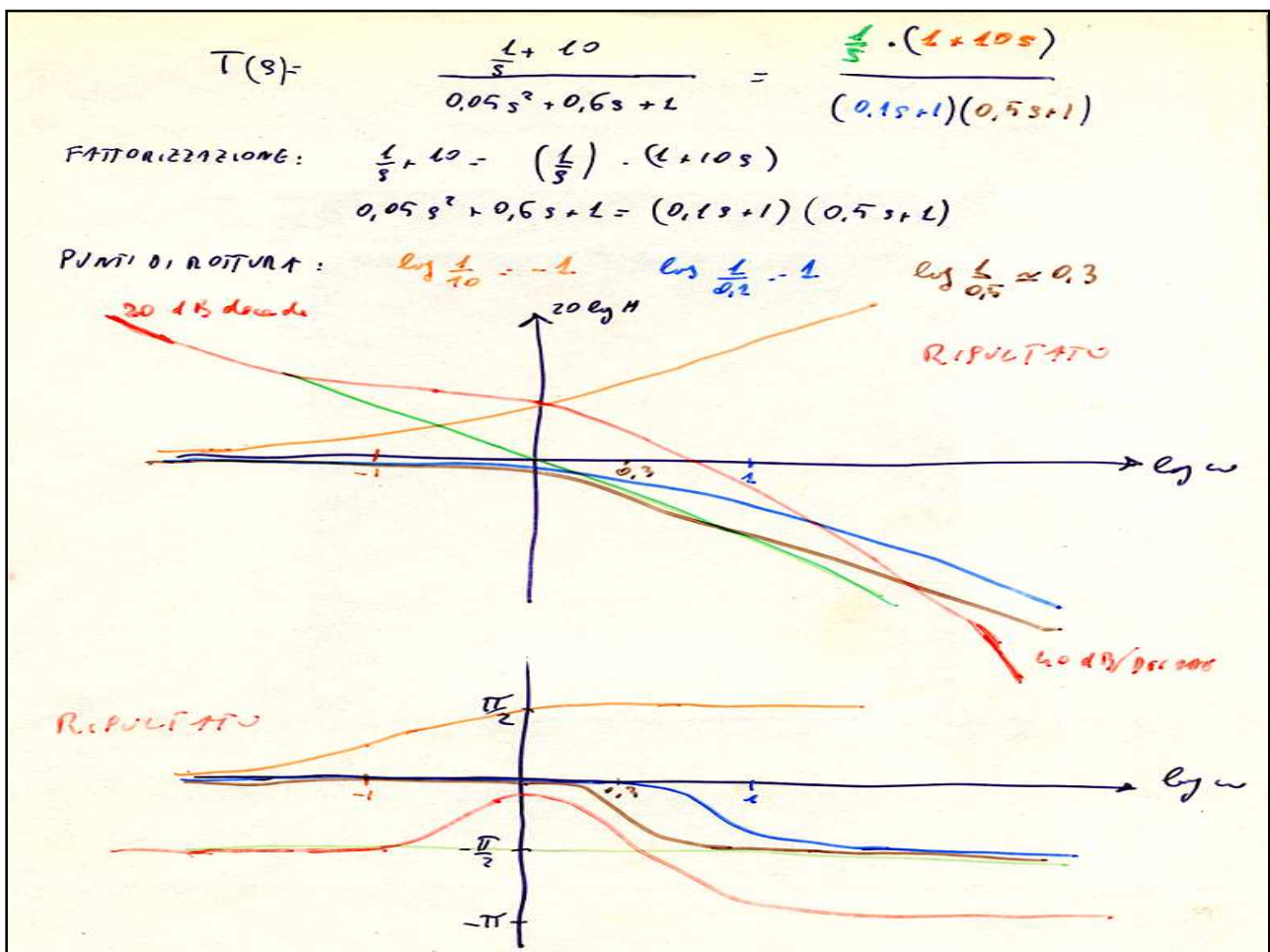
$$T_2 = 0,2 \quad \sqrt{T_2} = 5 \text{ RAD/S}$$



$$\frac{g_u}{g_i}(i\omega) = \frac{0,016 \overset{1}{(0,33 \overset{2}{i\omega + 1})}}{\underset{3}{(0,05 i\omega + 1)} \left[ \left(\frac{i\omega}{80}\right)^2 + \frac{0,5i\omega}{80} + 1 \right] \underset{4}{}}$$



Esempio di tracciamento del diagramma asintotico e corretto per un particolare sistema.



DIAGRAMMI LOGARITMICI O DI BODE

$$T(i\omega) = \frac{G_u(i\omega)}{G_i} = M(\omega) \cdot \exp(i\phi(\omega))$$

$$M(\omega) = |T(i\omega)| = \sqrt{R^2(\omega) + I^2(\omega)}$$

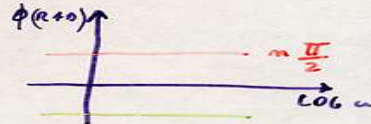
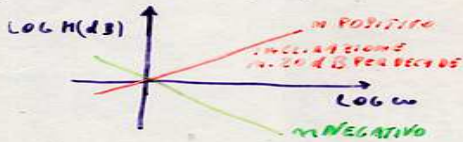
$$\phi(\omega) = \arctg \frac{I(\omega)}{R(\omega)}$$

SI ESEGUE LA FATTORIZZAZIONE DI  $T(i\omega)$

1) TERMINI COSTANTI :  $\frac{b_0}{a_0} = k =$  SENSIBILITA' STATICA



2) TERMINI  $(i\omega)^m$



3) TERMINI  $(1 + i\omega T)^{-1}$  : PUNTI DI ROTTURA :  $\text{LOG } \omega = \text{LOG}(1/T)$

